

### Exercise Sheet 1

**Exercise 1:** Prepare a Matlab script that computes

- Lagrange polynomial  $\Pi_n$  in standard form,
- Lagrange polynomial  $\Pi_n$  with Newton Divided Difference,
- Piecewise Lagrange approximation.

INPUT:

- $a, b$  the boundary of the domain,
- $N$  number of subintervals of the partition of  $[a, b]$ ,
- $n$  degree of the polynomial (for piecewise approximation)
- $f$  the function you want to interpolate.

OUTPUT:

- The error in  $\| \cdot \|_\infty$  between the function  $f$  and the polynomial approximation.,
- the plot of the polynomial approximation and the exact solution with different colors,
- the plot of the error when increasing the degree  $n$  of the polynomial.

You might test your code on the following functions

- $f(x) = \sin(3x) \quad 0 \leq x \leq 2\pi,$
- $f(x) = \log(x) \quad 1 \leq x \leq 1.5,$
- $f(x) = e^x, \quad 0 \leq x \leq 1,$
- $f(x) = x^5 + 3x^4 + 2x^3 - x^2 - 5 + 1, \quad -5 \leq x \leq 5,$
- $f(x) = \frac{1}{1+x^2}, \quad -5 \leq x \leq 5,$
- $f(x) = |x|, \quad 0 \leq x \leq 5, \text{ and } -3 \leq x \leq 2,$
- $f(x) = \sin(x^2), \quad -5 \leq x \leq 5,$
- $f(x) = |\sin(x^2)|, \quad -5 \leq x \leq 5,$
- $f(x) = \operatorname{sgn}(x), \quad -5 \leq x \leq 5,$