Department of Mathematics
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Exercise Sheet 1

Exercise 1: Solve the following ODEs:

$$\begin{aligned} \dot{y}(t) &= -4y + 2, \\ y(0) &= 5. \end{aligned}$$

$$\dot{y}(t) = 2y + 5.$$

Exercise 2: Find constants $a, b \in \mathbb{R}$ such that

$$y(t) = (t+3)e^{2t}$$

solves

$$\begin{aligned} \dot{y}(t) &= ay + e^{2t}, \\ y(0) &= b. \end{aligned}$$

Exercise 3: Solve the following ODEs:

$$\begin{aligned} 2ty - \dot{y} &= 0, \\ y(0) &= 3 \end{aligned}$$

$$\frac{\dot{y}}{(t^2+1)y} = 4t$$

$$\dot{y} = y + \frac{3}{y^2}$$

$$ty + \sqrt{1+t^2}\dot{y} = 0$$

Exercise 4: Use the Picard iteration to find the solution of

$$\begin{aligned} \dot{y}(t) &= 5ty \\ y(0) &= 1 \end{aligned}$$

Exercise 5: Find the domain where the solution of the IVP below is well-defined

$$\begin{aligned} \dot{y}(t) &= -\frac{4t}{y}, \\ y(0) &= y_0 > 0 \end{aligned}$$

$$\dot{y}(t) = 2ty^2,$$
$$y(0) = y_0 > 0$$

Exercise 6: Solve the following ODEs:

$$y'' - 8y' + 25y = 0,$$
$$y(0) = 2, y'(0) = 2.$$

$$y'' - 6y' + 9y = 0,$$
$$y(0) = 1, y'(0) = 2$$

$$y'' + y' = 3\sqrt{e},$$
$$y(0) = 4, y'(0) = 3.$$

$$y'' + 5y' - 9y = x^2 + cos(2x).$$

Exercise 7: Check if the following functions are linearly independent:

- $f(t) = \sin(t), \quad g(t) = \cos(t);$
- $f(x) = x, \quad g(x) = xe^x;$
- $f(\theta) = \cos^2(\theta), \quad g(\theta) = 1 + \cos(2\theta).$

Exercise 8: Find every solution y of the ODE

$$\dot{y} = \frac{t^2 + y^2}{ty},$$

$$(t^2 + 2ty(t))\dot{y}(t) = y^2(t),$$
$$y(1) = 1.$$