

Exercise Sheet 2

Prepare a Matlab script that computes the numerical approximation of the Cauchy problem

$$\begin{aligned}\dot{y}(t) &= f(t, y(t)), \quad t \in I \\ y(t_0) &= y_0\end{aligned}$$

utilizing θ - method with particular focus on $\theta = \{0, 0.5, 1\}$ and Heun's method.

INPUT:

- f , the function which describes the vectorial field
- t_0, y_0 initial conditions;
- T final time
- Δt temporal step size;

OUTPUT:

- Absolute error when the solution is known.
- the plot of the numerical approximation and the exact solution (when known) with different colors,
- Order of convergence of the scheme.

You might test your code on the following ODEs

- $\dot{y} = -y \log y, y(0) = 0.5,$ exact solution $y(t) = e^{-e^{\log \log(2)-t}}$
- $\dot{y} = -e^{-(t+y)}, y(0) = 1,$ exact solution $y(t) = \log(e + e^{-t} - 1)$
- $\dot{y} = y(1 - y), y(0) = 0.5,$ exact solution $y(t) = e^t / (1 + e^t)$
- $\dot{y} = 16y(1 - y), y(0) = 1/1024,$ exact solution $y(t) = e^{16t - \log 1023} / (1 + e^{16t - \log 1023})$
- Harmonic oscillator
$$y'' = -\omega^2 y, y(0) = x_0, y'(0) = y_0 \omega,$$

exact solution: $y(t) = x_0 \cos(\omega t) + y_0 \sin(\omega t)$
- $y'' = -\omega^2 y - \alpha y', y(0) = x_0, y'(0) = y_0 \omega$
- $y'' = -\omega^2 y + f(t), y(0) = x_0, y'(0) = y_0 \omega,$ with $f(t) = \{\sin(t), t \sin(t), t^2\}$.
- Pendulum with friction

$$\begin{aligned}\theta' &= -\theta' - \sin \theta, \\ \theta' &= \theta'_0, \theta(0) = \theta_0\end{aligned}$$

i) Lotka-Volterra

$$\begin{aligned}y_1' &= \alpha y_1 - \beta y_1 y_2 \\y_2' &= -\gamma y_2 + \delta y_1 y_2\end{aligned}$$

For example: $\alpha = 0.25, \beta = 0.01, \gamma = 1, \delta = 0.01, y_1(0) = 80, y_2(0) = 30, t_0 = 0, T = 30$.

j) Van der Pol

$$\begin{aligned}y_1' &= y_2 - f(y_1) \\y_2' &= -y_1\end{aligned}$$

with $f(x) = x^3 - x, y_1(0) = x_1, y_2(0) = x_2$.