Department of Mathematics 2018.2
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Exercise Sheet 3

Exercise 1: Prepare a script that computes the polynomial interpolation based on Chebyshev nodes and the gaussian quadrature approximation with Chebyshev's weiths

INPUT:

- a, b the boundary of the domain,
- n degree of the polynomial
- f the function you want to interpolate.

OUTPUT:

- The error in $\|\cdot\|_\infty$ between the function f and the polynomial approximation,
- the plot of the polynomial approximation and the exact solution with different colors,
- the approximate value of $\int_a^b f(x)dx$,
- absolute error of the quadrature approximation when the value is known

You might test your interpolation code on the following functions

a) $f(x) = \sin(3x) \quad 0 \leq x \leq 2\pi,$

b) $f(x) = \log(x) \quad 1 \leq x \leq 1.5,$

c) $f(x) = e^x, \quad 0 \leq x \leq 1,$

d) $f(x) = \dfrac{1}{1+x^2}, \quad -5 \leq x \leq 5,$

e) $f(x) = |x|, \quad 0 \leq x \leq 5,$ and $-3 \leq x \leq 2,$

f) $f(x) = \sin(x^2), \quad -5 \leq x \leq 5,$

g) $f(x) = |\sin(x^2)|, \quad -5 \leq x \leq 5,$

h) $f(x) = sgn(x), \quad -5 \leq x \leq 5,$

You might then test your quadrature approximation on the following functions

a) $\int_0^{2\pi} \sin(x) = 0,$

b) $\int_0^{2\pi} xe^{-x}\cos(2x) = \dfrac{3(e^{-2\pi}-1) - 10\pi e^{-2\pi}}{25},$

c) $\int_{-1}^0 |x|\, dx = 0.5 \quad \int_{-1}^1 |x|\, dx = 1,$

d) $\int_{-2}^2 x\sin(x)\, dx,$

e) $\int_0^1 x^{5/2}\, dx = 2/7,$

f) $\int_0^1 e^{-x}x^{-1/2}\, dx.$